Empirical Methods HW3

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**Problem 2.1**

*et* = *et*−1 + *xt* =⇒ *xt* = *et* − *et*−1 *xt* = *φxt*−1 + *εt yt* ≡ *et* − *et*−4

*xt* = *et* − *et*−1 = *φxt*−1 + *εt* = *εt, for φ* = 0 *xt* = *εt* = *et* − *et*−1 *εt*−1 = *et*−1 − *et*−2 *εt*−2 = *et*−2 − *et*−3 *εt*−3 = *et*−3 − *et*−4 *yt* = *εt* + *εt*−1 + *εt*−2 + *εt*−3 = *et* − *et*−4

*E*[*yt*]=0+0+0+0=0*, Find Cov*(*yt,yt*−*j*) *for j* = 0*,*1*,*2*,*3*,*4*,*5 *j* = 0*,Cov*(*yt,yt*) = *V ar*(*yt*) = *ε*2*t* + *ε*2*t*−1 + *ε*2*t*−2 + *ε*2*t*−3 =1+1+1+1=4

*j* = 1*,Cov*(*yt,yt*−1) = *E*[*ytyt*−1] − *E*[*yt*]*E*[*yt*−1] = *E*[*ytyt*−1] − 0 = *E*[(*εt* + *εt*−1 + *εt*−2 + *εt*−3)(*εt*−1 + *εt*−2 + *εt*−3 + *εt*−4)]*, E*[*εiεj*]=0 ∀*i* = *j* =⇒ *E*[*ε*2*t*−1 + *ε*2*t*−2 + *ε*2*t*−3]=1+1+1=3

*Similarly, j* = 2*,Cov*(*yt,yt*−2) = *E*[(*εt* + *εt*−1 + *εt*−2 + *εt*−3)(*εt*−2 + *εt*−3 + *εt*−4 + *εt*−5)] = *E*[*ε*2*t*−2 + *ε*2*t*−3]=1+1=2

*Again, j* = 3*, Cov*(*yt,yt*−3) = *E*[*ε*2*t*−3]=1

*j* = 4*,Cov*(*yt,yt*−4)=0*, j* = 5*,Cov*(*yt,yt*−5)=0

**Problem 2.2**

The autocovariance drops to 0 after 3 lags. So the for every value after 3 it isn’t correlated to *yt*. With *φ* = 0 We get the AR(0), this is a MA(3) then. So we have a ARMA(0,3).

**Problem 3**

1.

*V ar*(*Ret*+1) = *β*2*V ar*(*xt*) + *V ar*(*εt*+1)=1 ∗ (0*.*05)2 + 0*.*152 = 0*.*025 = √0*.*025 = *.*15811

1

2.*R*2 = *ρ*2 = (*Cov*(*RσRet*+1*t*+1*eσx,xt t*)

)2 = (*Cov*(*βxσRt et*+1+ *σεxt*−1*,xt t*)

)2 = ( *σV Rar*(*xet*+1*σt*)

*xt* )2 = *V V ar*(*Rar*(*xet*+1*t*)

) = 0*.*052

0*.*025 = 0*.*1

3. Sharpe Ratio = *E*[*Rt*+1*eσmkt* ]

= √0*.*05 0*.*025 = 0*.*31622 4.

*γ* = 409 *αt* = *E*[*xt*]

*γσt*2[*Rt*+1*e*] = *E*[*xt*]

*γ*(*σt* 2[*xt*] + *σt*2[*εt*+1]) *So, if x* = 0*, then αt* = 0

*positive* =0 =⇒ *Sharpe Ratio* = 0 *if x* = *.*1*,αt* = *.*1

409 (*.*052 + 0*.*152)

=1 =⇒ *Sharpe Ratio* = *.*1111 *.*1

= *.*9

5.a) Below is the Expected value of the return gamma <- 40**/**9 xt\_1 <- 0 xt\_2 <- 0.1 cond\_var <- .15**^**2 E\_x2 <- **var**(**c**(0,.1)) **+** 0.05**^**2 uncond\_ret <- 0.5**\***xt\_1 **+** 0.5**\***xt\_2 alpha = (uncond\_ret **/** (gamma **\*** (cond\_var **+ var**(**c**(0,.1))))) E\_alpR <- alpha **\*** uncond\_ret E\_alpR

## [1] 0.02045455

b) The output below is the unconditional standard deviation var\_alpR <- (E\_x2 **+** cond\_var **-** uncond\_ret**^**2) **\*** alpha**^**2 **sqrt**(var\_alpR)

## [1] 0.06784005

c)Below is the sharpe value E\_alpR **/ sqrt**(var\_alpR)

## [1] 0.3015113

d) i.Below is the implied Rˆ2 x <- **c**(**-**0.05,.15) E\_x <- **mean**(x) var\_x <- **var**(x) var\_R <- var\_x **+** cond\_var E\_xSq <- var\_x **-** E\_x**^**2 R\_Sq <- var\_x **/** var\_R R\_Sq

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## [1] 0.4705882

ii. alpha1 = (E\_x **/** (gamma **\*** (cond\_var **+** var\_x))) var\_alpR1 <- (E\_xSq **+** cond\_var **-** E\_x**^**2) **\*** alpha1**^**2 (alpha1 **\*** E\_x) **/ sqrt**(var\_alpR1)

## [1] 0.2581989

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